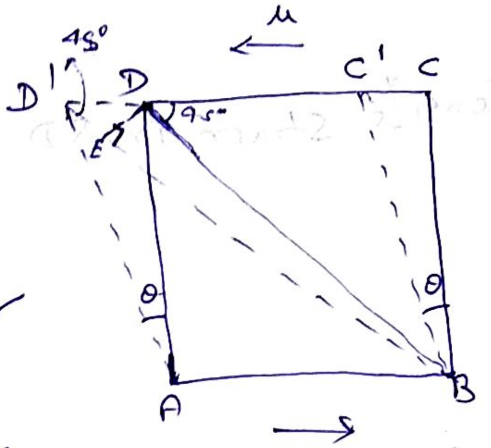
Diagonal Strains:



In earlier lecture notes we have studied diagonal stresses produced by a shear on a square block.

We have seen that the set of shear stresses μ on the faces AB, CD and the faces AD and CB are equivalent to a tensile stress (of intensity μ) along the diagonal BD and a compressive stress (of intensity μ) along the diagonal AC. Here we will obtain the expression for direct (tensile and compressive) strains of diagonal AC and BD.

we define $\sigma \rightarrow$ Poisson's ratio

$Y \rightarrow$ Young's modulus of the material considered,

Next, the tensile strain in diagonal BD due to tensile stress μ along BD $= \frac{\mu}{Y}$

Tensile strain in BD due to compressive stress μ along AC $= \sigma \cdot \frac{\mu}{Y}$

$\Rightarrow$ Total tensile strain along BD $= \frac{\mu}{Y} + \frac{\sigma\mu}{Y}$

$= \frac{\mu}{Y}(1+\sigma)$ ——— ①

Similarly we obtain the total compressive strain along diagonal AC $= \frac{\mu}{Y}(1+\sigma)$ ——— ②

Next, we will show that the total tensile strain in BD can be expressed as the half of the shear strain.

$\therefore$ Tensile strain in BD = $\dfrac{\text{Increase in length}}{\text{Natural length}}$

$= \dfrac{BD' - BD}{BD}$

See Fig. DE is perpendicular drawn on the BD'.. length DD' is very small ~~...~~ ~~...~~ Therefore angle DBD' will be very small. and we can take $BD = BE$

and angles $\angle CDB = \angle C'D'B = 45^\circ$

$\Rightarrow \angle DD'E = 45^\circ$

length $D'E = DD' \cos(DD'E)$

$= DD' \cos 45^\circ = \dfrac{DD'}{\sqrt{2}}$

In $\triangle ABD$, $BD = \sqrt{AB^2 + AD^2} = \sqrt{2}\, AD$ $\{\because AB = AD\}$

$\therefore$ Tensile strain in BD $= \dfrac{BD' - BD}{BD} = \dfrac{BD' - BE}{BD} = \dfrac{D'E}{BD}$

$= \dfrac{DD'/\sqrt{2}}{\sqrt{2}\, AD} = \dfrac{1}{2} \dfrac{DD'}{AD}$

$\because \dfrac{DD'}{AD}$ = shear strain

Therefore, Tensile strain in diagonal BD = $\dfrac{1}{2}$. shear strain